UTSLesson Plan Format

Key Learning Area/s: Mathematics	**Class/Year: 9**	**Date:** **Time: Start ___________ Finish ___________**

Lesson Goal: Describe the results of two- and three-step chance experiments, ~~with and without replacement~~, assign probabilities to outcomes, and determine probabilities of events; investigate the concept of independence (ACMSP246)

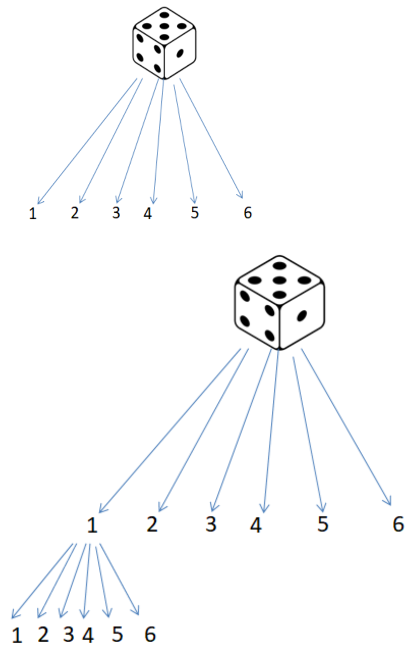
Recent Prior Experience:

- the concept of theoretical probability
- the definition of sample space and event
- intuitive understanding of the concept "likelihood"
- Calculate probability for one-step chance experiments
- Differentiate between compound and simple events and their probabilities

Language: Probability, Independent, Event, Outcome, Weighting

Syllabus Outcome(s):	**Learning Intentions:**	**Success (assessment) criteria:**
Stage 5 Statistics and Probability: Probability of Compound Events <u>Outcomes</u>: A student calculates relative frequencies to estimate probabilities of simple and compound events (MA5.1-13SP) <u>Content</u>: Describe the results of two- and three-step chance experiments, with and without replacement, assign probabilities to outcomes, and determine probabilities of events; investigate the concept of independence (ACMSP246) ● Calculate probabilities for intersections of simple independent events, including sample spaces with unevenly weighted outcomes	*<u>By the end of this lesson, the students will</u>:* ● Differentiate between dependent and independent events ● Use the formula P(A and B)=P(A)×P(B) in two-step chance experiments ● List outcomes of two-step experiments and the related outcomes of the intermediary steps ● Demonstrate examples where events can be unevenly probable in a list of outcomes	● A student can explain intuitively (non-rigorously) why P(A and B)=P(A)×P(B) ● A student can list the outcomes for two and three step experiments and the steps leading to them ● A student can assign probabilities to the outcomes of two and three step experiments through explanation of the steps **Assessment** ● A student can show the probability of rolling 1 twice on a standard die is 1 in 36, both by applying formula and listing equally likely outcomes ● A student can predict that the sum of two dice rolls is most likely to be 7 and explain why. ● A student can differentiate between equally and unequally weighted outcomes

Lesson Content (*What* is Taught):	**Timing** *(mins)*	**Teaching Strategies / Learning Experiences:**	**Resources and Organisation:**
INTRODUCTION (link to previous understanding, skill attitudes. Explain new understanding, skills, attitudes to be investigated)			
Revision: ● Single step experiments ● Examples of sample space	5 minutes	What are the outcomes if you roll a standard dice? Are all the outcomes equally likely? How can you know? What if I repaint the 6-dots-side to 5 dots? Class-wide conversation, open strategy and targeted sharing	On whiteboard, after student response: {1,2,3,4,5,6} P(1) = ? P(2) = ? (Ans: $\frac{1}{6}$) {1,2,3,4,5}: P(5) = ? (Ans: 2/6)
DEVELOPMENT (the body of the lesson)			
Students: ● find outcomes of two rolls added together in case where each outcome is equally likely ● List steps to get each outcome ● Assign probabilities intuitively (not using formula)	3 minutes	Teacher-led exposition of task: display die with sides 1, 2, 5, 12 (small numbers that generate different sums in pairs)	One 4-sided die Paper for students to make dice (optional)
	5 minutes	If I roll the die twice and add the results, what's the chance I'll get seven? Work in pairs Ans: $\frac{1}{8}$, the result of (2,5) or (5,2)	
Students: ● list the outcomes of two rolls of a 6-sided die ● predict the mostly likely sum of two rolls ● use real dice to test their hypothesis ● Compare their results with a partner ● Connect concept of mode and probability	3 minutes	Teacher-led transition to six-sided dice What's the sample space of two rolls?	6-sided dice for student use
	5 minutes	Can you predict the sum you'll see most often if you try rolling your die? Ans: 7 (note, this is not related to 7 from earlier) Revise concept of mode, link to "most likely" (Analogy of numbers to coloured balls) Work solo then compare in pairs	
Students: ● informally explain how independence of events relates to their probabilities ● learn the term independence ● first encounter with formula P(A and B)=P(A)×P(B) ● calculate the probabilities of different sums	7 minutes	Targeted strategy sharing for how to determine the probabilities of different sums from two rolls Student-led discussion of how to combine probabilities of two rolls (non-rigorous). Use tree to show how second roll expands each branch into a second tree. Introduce concept of independence, link to real world contexts Introduce formula P(A and B)=P(A)×P(B)	On whiteboard: tree diagram of one and two dice rolls (example of first and beginning of second below) List of outcomes: {2,3,4,5,6,7,8,9,10,11,12} (Dice image credit in references, tree drawn in Microsoft Powerpoint)

		Give examples of dependent events where formula does not apply, e.g. P(1 and not(6)) on dice roll	
	5 minutes	What is the probability of the sum being 7? What about 6 or 8? *(And so on, depending on time)* Start with students calling out pairs that yield 7. Work in pairs to do calculation using formula.	On whiteboard, after student input: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)
		Differentiation*(supporting students who need more help, or challenging students who need to be extended)* Easier: Demonstrate examples of unequal outcomes on a weighted die. Draw decision trees of coin flips and have them work those first before approaching dice decision trees. Harder: What's the shape if you graph the outcomes vs. their occurences? (Try graphing with GeoGebra Math Apps). How does the curve change if you add friends' data? What's the shape if you graph the outcomes vs. their theoretical probabilities? If your first roll is a 1, is it less likely you'll roll a 1 again? (Ans: Bell curve and no, that's called the Gambler's Fallacy)	A weighted die Useful website (for teacher use only): https://anydice.com/

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CLOSURE (Allow review of new understanding, skill, attitudes)			
	5 minutes	What did you notice about the probabilities of the different sums? *Things to highlight:* ● Definition of independent events ● Formula only applies if events are truly independent ● Can also use decision trees if rolls are fair, i.e. outcomes equally likely ● Real world example: basketball teams' chances at winning multiple games (not 50-50) Isn't it a "coincidence" that 7, the most probable outcome is also the average, mode and median of the outcomes? Why might this be? (parting question)	

Evaluation:

Any safety issues to be considered: Students may throw dice at each other, four-sided dice is a stepping hazard

Lesson Play

T:	*So I've got this four-sided die, what's this called?*
S1:	*That's a triangular prism. No, a pyramid. A triangular pyramid.*
T:	*He's on fire today. That's right, it's a triangular pyramid. It's actually a regular tetrahedron, they're all equilateral triangles.* *So what I'm going to do is write some numbers on the faces. I'll put 1 here, and 2 here, and 5 on this one, and, oh, 12 on the last face.* *I'm going to roll the die twice, and each time I'm going to make a note of the number on the bottom.* *I want to know, what's the chance, if I add them up, what's the chance I'll get a total of seven?*
S1:	*(to S2) We need to find all of the possible answers.*
S2:	*OK. So 1 and 2 is 3. 2 and 5 is 7. We can get, um, 3, 7, 17, ... oh. We can get 6. Let's write this down.*
S1:	*2, 3, 6, 13. That's if one of them is a 1.*
S2:	*OK you can get a 4, with 2 and 2. And 7, 14. And 17, that's 5 + 12.*
S1:	*OK that's 2, 3, 4, 6, 7, 13, 14, 17. There's, um, 8 answers.*
S2:	*So the chance of a total of 7 is one-eighth.*
T:	What happens if you roll 5 both times?
S1:	You get 10? Wait, that’s not in our list.
T:	You’ve got 2 and 4 though. Is there any outcome you’re missing other than 10?
S1:	Yeah, I guess we missed a few. 5 and 5 is 10, 12 and 12 is 24.
S2:	So 10 answers, the chance of 7 is one-tenth.
T:	If there are 10 outcomes, is the probability of one of them *always* one-tenth?
S2:	Er, I think so.

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T:	How many ways are there to get each outcome?
S2:	Whaddya mean, Miss?
T:	Well, how do you get 7 as a sum of rolls?
S1:	You add 2 and 5.
S2:	You have to roll 2 and 5.
T:	That's right. Now if you roll a 2 and then a 5, you get a 7. What happens when you roll a 5 *then* a 2?
S2:	You get 7.
S1:	Oh, so the order doesn't matter.
T:	Not with sums, right?
S1:	Yeah, they're… commutative?
T:	That's the word. So how many ways are there to get 7 if you can roll either 2 or 5 first?
S2:	Two? But if order doesn't matter, that's really one, isn't it?
T:	Getting a 2 or 5 is still two different rolls of the die, so you're right, it's two.
S1:	Wouldn't each outcome also have two ways though?
T:	Well, let's look at some of the other sums. Are there any pairs of numbers other than 5 and 2 that sum to 7?
S1:	On the die? *checks* I don't reckon, no.
T:	Does each sum have two different numbers that add up to it?
S1:	3 is 1 and 2, 4 is 2 and 2, wait, those are the same.
T:	Just so. What does that mean for us? How many ways are there to get 4?
S2:	You have to roll 2 both times, there's no other way.
T:	Spot on! So now you've found an outcome that only has one sequence that results in it.
S2:	Uh, I guess.

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T:	What does that mean for the probability of getting 4 as the sum?
S1:	It’s less likely?
T:	Right. How much less likely are you to get 4 than 7?
S1:	Half as likely? Since 7 has two ways and 4 has one?
T:	You’ve nailed it. So what’s the probability of getting 7?
S1:	We said one-tenth. But if 4 is half as likely, we’d have to, um. So 4 is half as likely as the other outcomes, that means… *starts scribbling algebra*
T:	Is 4 the only outcome that has only one sequence?
S1:	I dunno.
S2:	Gotta be 10, you said it’s 5 and 5. And 12 + 12, 24. The last side is 1, so 2.
T:	You’re right. I like how you looked at the sides of the die, that was a good way to check what you know. So you have 2, 4, 10 and 24 as less likely. Now let’s try doing algebra, <S1’s name>.
S1:	So we’ve got the probabilities sum up to 1. If the probability of getting 7 is x...
S2:	We’ve got ½ x for 2, 4, 10, 24 and x for all the others. That’s uh, $2x$ plus..
S1:	Six others. $6x$.
S2:	So $8x = 1$. Hey, the probability of getting 7 *is* one-eighth?
S1:	That’s what we had in the beginning!
T:	Funny coincidence, that. Good job, both of you, you’ve really got it straight now. Good job reasoning it out.

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Justification

My fundamental approach draws from the Socratic Method, which has been linked by research to models of learning. (Yengin & Karahoca, 2012). The Socratic Method lends itself to explaining mathematical reasoning. In fact, the original dialogue discussed geometry. (Plato et al., 2004). By asking more general questions, which indicate concepts rather than procedures, the Socratic Method dovetails with Watson's focus on encouraging probabilistic reasoning in a conceptual sense, rather than focusing on formulae. (J. Watson, 2005, as cited in Goos et al., 2016, p. 278). This focus encourages relational, rather than instrumental, understanding. (Skemp, 1976).

I begin the lesson plan with open strategy sharing, suitable for a revision exercise because it solicits general feedback which allows me to gauge overall understanding. I then narrow the scope of solution approaches through targeted sharing, which sets the students up for success when attempting the question I set them. (Hintz & Kazemi, 2014). Students work collaboratively (in pairs) to help pool knowledge and reduce misunderstandings while increasing their confidence in concept. (Berčíková, 2007, p. 12; HITS, 2020). Showing both listing outcomes and using the formula for independent events as equivalent methods is also important, because it demonstrates the existence of multiple methods. This reduces a student's fear of doing a problem "the wrong way". (Lee, 2006, p. 19)

Students using real dice incorporate a concrete element of experimentation, as opposed to the more abstract calculations they conduct before and after. This is motivated by the success of school statistical experiments such as testing loopy airplanes. (Makar, 2013, pp. 35-37). Discussion of most likely outcomes also explicitly references modes in order to link statistical concepts to probability, as emphasized in Makar's research. (2013, pp. 37-39). This additionally demonstrates the use of a measure of central tendency, intended to help students view data in aggregate. (Goos et al., 2016, 279-80). In another use of supplementary materials, I provide a visual aid by use of a tree diagram, recommended by Goos et al. (2016, p. 282). It demonstrates how experiments with multiple independent steps are related to the individual steps, which are themselves probability experiments.

I have chosen to limit the use of technology (ICT) in the lesson, because we are discussing a more basic conceptual understanding of 2-step and 3-step events, where there is a risk that students will focus on procedure and merely obtaining the "right" answer. (Skemp, 1976). Goos et al. discuss how students may overly rely on technology to compensate for a lack of mathematical understanding (2016, pp. 95-96). To avoid this, I defer use of ICT to later lessons or in this plan, a differentiation activity, for students who have already demonstrated a relational understanding.

In my lesson play, I assume that the two students would merely take the different sample size (ten outcomes) as input for their algorithm rather than rethink their approach. This expectation, i.e., that students will do the minimum adjustment needed to their thought process to account for new information, comes from research on the laziness of thinking systems. (Kahneman, 2013, pp. 31, 35, 39-49).

The lesson play makes use of several instances of praise for method, rather than for results. The response to incorrect answers is also critical to the development of reasoning, particularly being careful to not make any suggestion regarding the students' inherent capacity for reasoning. This draws from Carol Dweck's research on growth mindset and motivation. (Dweck & Leggett, 1988). Zazkis et al. also emphasize the opportunity for teachers to help students develop mathematical reasoning rather than directly contradicting wrong answers. (2013, pp. 12-13).

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Overall, I believe in letting students discover concepts through as much of their own mental effort as possible. There is a sense of achievement when one has a breakthrough in understanding a concept, and that comes through following your own reasoning rather than someone else's. Strong emotion is a powerful aid to memory (Tyng et al., 2017), and I would prefer students have positive emotions associated with school rather than negative.

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